Determinants

Multiple Choice Questions

Q: 1 **U and V are two non-singular matrices of order n and k is any scalar.**

Given below are two statements based on the above information - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion (A) **: det(k U) = $k^n \times$ det(U)**

Reason (R) **: If W is a matrix obtained by multiplying any one row or column of V by the scalar k, then det(W) = $k \times$ det(V).**

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is true but (R) is false.
4. (A) is false but (R) is true.

Free Response Questions

Q: 2 [1]

Show that for any positive integer n, $\begin{vmatrix} (n-1)! & n! \\ -n & n \end{vmatrix} = (n+1)!$.

(Note: n! denotes factorial n.)

Q: 3 **Given below is a system of linear equations in three variables.** [2]

$tx + 3y - 2z = 1$
$x + 2y + z = 2$
$-tx + y + 2z = -1$

For what value of t, will the system fail to have a unique solution? Show your work.

Q: 4 **P(k, -1), Q(3, -5) and R(6, -1) are vertices of ΔPQR. The lengths of side QR and altitude PS are 5 units and 4 units respectively.** [2]

Use determinants to find the value(s) of k. Show your work.

Q: 5 [2]

If A = $\begin{pmatrix} 4 & -8 \\ 12 & 16 \end{pmatrix}$ and B = $\begin{pmatrix} -16 & -8 \\ 12 & -4 \end{pmatrix}$,

identify what type of matrix is (Adj A) $\times$ (Adj B).

Show your work.

Q: 6 **[3]**

Consider the matrix $X = \begin{bmatrix} p & -2 & -3 \\ -2 & 2 & 6 \\ 1 & 3 & q \end{bmatrix}$, where p and $q \in$ R.

The co-factor of element 6 is (-11) and the minor of element 2 is 0.

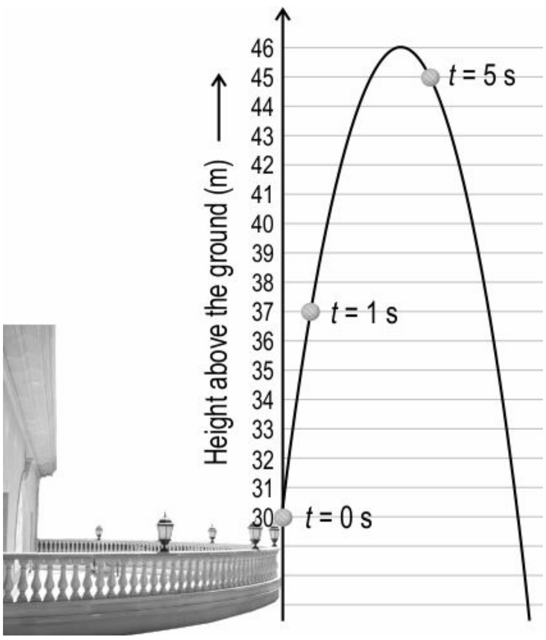
Find the values of *p* and *q*. Show your work.

Q: 7 **Without expanding the determinant, show that Δ is a perfect square for any integer values of *a* , *b* and *c* .** **[5]**

$$\Delta = \begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

Q: 8 **A ball is thrown from a balcony. Its height above the ground after *t* seconds is given by $h(t) = pt^2 + qt + r$, where *p* , *q* , and *r* ∈ R, and *h* (*t*) is in meters.** **[5]**

Shown below is the trajectory of the ball with its height from the ground at *t* = 0 s, *t* = 1 s and *t* = 5 s.



Solve for *p* , *q* and *r* and find *h(t)* using the matrix method. Show your work and give valid reasons.

Q: 9 **A and B are invertible matrices of same order such that:** **[5]**

$$(AB)^{-1} = \frac{1}{48}\begin{bmatrix} 0 & 0 & 16 \\ 0 & 24 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$\text{and } A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Find B. Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	1

Q.No	What to look for	Marks
2	**Expands the determinant as:** $n(n-1)! + n(n!)$	0.5
	Simplifies the above expression as: $n! + n(n!) = n!(n+1) = (n+1)!$	0.5
3	**Writes the coefficient matrix and finds its determinant as:** $\begin{vmatrix} t & 3 & -2 \\ 1 & 2 & 1 \\ -t & 1 & 2 \end{vmatrix} = t(3) - 3(2+t) - 2(1+2t) = -4t - 8$	1
	Solves the equation -4 t - 8 = 0 for t and writes that for t = (-2), the system fails to have a unique solution.	1
4	**Uses QR and PS to find the area of the triangle as 10 sq units. The working may look as follows:** $\frac{1}{2} \times 5 \times 4 = 10$	0.5
	Equates the area found to that using determinants as follows: $\frac{1}{2}\begin{vmatrix} k & -1 & 1 \\ 3 & -5 & 1 \\ 6 & -1 & 1 \end{vmatrix} = 10$	0.5
	Expands the above determinant to write the equation as: $\frac{1}{2}\left\| k(-5+1) + 1(3-6) + 1(-3+30) \right\| = 10$ $\left\| -4k + 24 \right\| = 20$	0.5

Q.No	What to look for	Marks
	Solves the above equation to find the values of k as 1 or 11.	0.5
5	Finds (Adj A) as: $\begin{pmatrix} 16 & 8 \\ -12 & 4 \end{pmatrix}$	0.5
	Finds (Adj B) as: $\begin{pmatrix} -4 & 8 \\ -12 & -16 \end{pmatrix}$	0.5
	Finds the product of (Adj A) and (Adj B) as: $\begin{pmatrix} -160 & 0 \\ 0 & -160 \end{pmatrix}$	0.5
	Identifies the matrix (Adj A) × (Adj B) as a scalar matrix. (Award full marks if diagonal matrix is written instead of scalar matrix.)	0.5
6	Frames an equation in p using the co-factor of element 6 as $A_{23} = (-1)^{2+3} \begin{vmatrix} p & -2 \\ 1 & 3 \end{vmatrix} = -3p - 2 = -11$	1
	Solves the above equation to find the value of p as 3.	0.5
	Frames an equation in q using the minor of element 3 as $M_{22} = \begin{vmatrix} p & -3 \\ 1 & q \end{vmatrix} = \begin{vmatrix} 3 & -3 \\ 1 & q \end{vmatrix} = 3p + 3 = 0$	1
	Solves the above equation to find the value of q as (-1).	0.5

Q.No	What to look for	Marks
7	**Performs the row operation $R_1 \to R_1 - R_2 - R_3$ on Δ to get:** $$\Delta = \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$	1
	Takes (-2) common from the first row and performs the row operation $R_3 \to R_3 - R_2$ on Δ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2-b^2 & -a^2 & a^2 \end{vmatrix}$$	1
	Performs the row operation $R_2 \to R_2 - R_1$ on Δ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2-b^2 & -a^2 & a^2 \end{vmatrix}$$	1
	Performs the row operation $R_3 \to R_3 + R_2$ on Δ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix}$$	1

Q.No	What to look for	Marks
	Expands the determinant to get: $\Delta = (-2)[-c^2(b^2a^2) + b^2(-c^2a^2)]$ $= 4a^2b^2c^2$ $= (2abc)^2$ **Hence concludes that Δ is a perfect square for any any integer value of a, b and c.**	1
8	**Finds $r = 30$ by substituting $t = 0$ in $h(t)$.**	0.5
	Frames the following equations in p and q by substituting $t = 1$ and $t = 5$ respectively: $p + q = 7$ $5p + q = 3$	0.5
	Writes the above system of equations in the matrix form using AX = B as: $\begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$	0.5
	Finds |A| as -4 ($\neq 0$). Hence, writes that A^{-1} exists and the system has a unique solution.	0.5
	Finds *adj* A as: $\begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}$	0.5
	Finds A^{-1} using |A| and *adj* A as: $A^{-1} = \frac{1}{\|A\|} \times \text{adj } A$ $= \frac{1}{-4} \begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}$	1

Q.No	What to look for	Marks
	Writes that X = A^{-1} B and finds X as: $\begin{bmatrix} -1 \\ 8 \end{bmatrix}$ **Concludes that *p* = -1 and *q* = 8.**	1
	Writes $h(t) = -t^2 + 8t + 30$.	0.5
9	**Finds B^{-1} as $(AB)^{-1}$ A as follows:** $\frac{1}{48}\begin{bmatrix} 0 & 0 & 16 \\ 0 & 24 & 0 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $= \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{-1}{2} & 0 \\ \frac{-1}{8} & 0 & 0 \end{bmatrix}$	1.5
	Finds adj (B^{-1}) as follows: $adj(B^{-1}) = \begin{bmatrix} 0 & 0 & \frac{-1}{16} \\ 0 & \frac{1}{8} & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{8} & 0 \\ \frac{-1}{16} & 0 & 0 \end{bmatrix}$	1.5
	Finds det (B^{-1}) as 0 - 0 + 1($\frac{-1}{16}$) = $\frac{-1}{16}$.	1
	Finds the matrix B as follows: $B = \frac{1}{\frac{-1}{16}}\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{8} & 0 \\ \frac{-1}{16} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	1